Algebra 2: Unit 1 — Functions and Their Inverses

Topic	Instructional Foci
Topic 1: Inverse Relationships	In this topic, students develop conceptual understanding of functions and their inverses, informally defining *f* and *g* as inverse functions by: *If f and g are inverse functions, then when a value, n, is input into f and that result is then input into g, the final result is the original value, n. Additionally, when a value, n, is input into g and that result is then input into f, the final result is the original value, n.* Students use what they know about linear, quadratic, square root, and cube root functions from Algebra 1 to explore the relationships between pairs of inverse functions (e.g., quadratic and square root functions and cubic and cube root functions have inverse relationships). The use of known functions, including transformations and applications, allows for review while building new understanding. Honors students also verify that the graph of the inverse of a function can be determined by reflecting the graph of the function over the line $y = x$. *Note: The focus of this unit does not include composite functions. Composite functions will be used to further study inverse functions in future courses. In developing an understanding of the meaning of inverse functions, it is not acceptable for students to use a strategy to "switch" the x and y variables and then solve for y to determine an inverse function as this strategy promotes misconceptions.* **Concepts:** • Explore the nature of inverse operations. • Given graphs of relations, determine inverse relations using symmetry. • Determine connections among symbolic, graphic, and numeric representations of inverse relations. • Given a function, determine its inverse relation. • Given a function, determine its inverse relation.

Topic	Instructional Foci
Topic 2: Radical Expressions and Equations	Students apply their understandings of inverses to explore radical expressions, equations, and functions. Initially, the focus is on graphing radical functions and identifying key features. As students explore these functions and inverses, they will compare average rates of change. Students use a variety of strategies to solve simple radical and rational equations, and are able to move flexibly between radical and rational form. Finally, students apply understandings of radical functions to model and solve application problems. During this topic of study, honors students restrict the domain of a function so that its inverse is a function, verify that cubic functions have a point of symmetry, and identify situations that can be modeled by quadratic or square root functions. *Note: The focus of this unit does **not** include simplifying radicals. For example, $\sqrt{27}$ is an acceptable answer, and does not need to be expressed as $3\sqrt{3}$.* ___*Concepts:*___ • Graph radical functions expressed symbolically and show key features of the graph. • Graph radical functions expressed symbolically and show key features of the graph. • Use quadratic and radical functions to model and solve problems. • Calculate and interpret the average rate of change of a function over a specified interval. • Understand the relationship between rational exponents and radicals. • Solve simple radical equations. • Apply understandings of simple radical equations to solve problems.

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Topic 3: Exponential & Logarithmic Expressions, Equations, & Functions	In this topic, students extend what they have learned about inverse relationships to exponential and logarithmic functions (base 2, 10, and *e*). Students solve exponential and logarithmic equations graphically, numerically, and symbolically. This includes solving exponential equations by applying the definition of a logarithm. *Note: Students are not expected to utilize the properties of logarithms to evaluate expressions or solve equations in this course.* Students extend what they know about key features of graphs and transformations of graphs to exponential and logarithmic functions. Applications of these functions will be explored throughout this topic. Honors students also evaluate exponential functions at irrational values, determine effective rates of interest, graph using logarithmic scaling, estimate the rate of change of a function at a point, and solve more complex exponential equations. *Concepts:* • Extend understanding of exponential functions to non-integer domain values. • Graph exponential functions expressed symbolically and show key features of the graph. • Construct an exponential model given two points on an exponential graph. • Produce an equivalent form of an exponential expression to reveal percent rate of change. • Explore the effects of compounding on a percent rate of change. • Derive the number *e* and explore exponential functions that model continuous growth or decay. • Represent and solve exponential equations numerically and interpret the solutions. • Explore the inverse relationship between exponential and logarithmic functions • Use the relationship between exponential and logarithmic functions to evaluate expressions. • Use the relationship between exponential and logarithmic functions to evaluate expressions. • Graph logarithmic functions expressed symbolically and show key features of the graph. • Calculate and interpret the average rate of change of a function over a specified interval. • Identify the effect on exponential and logarithmic graphs by replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative). • Determine an equation for an exponential or logarithmic function from a table of values. • Determine the value of transformation constants given the graphs of exponential and logarithmic functions. • Represent and solve equations graphically and interpret the solution(s). • Solve exponential equations in the form $ab^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or *e*; evaluate the logarithm using technology. • Apply knowledge of exponential and logarithmic functions to a contextual situation.